CONFIDENCE INTERVAL

We present our currency exchange rate forecasts as confidence intervals, which show a potential range of future prices. The estimations are based on statistical properties of logarithmic returns that allow, after certain adjustment, application of classical analytical tools.

In this method we define logarithmic returns of an asset as

$$r_i = \ln\left(\frac{S_i}{S_{i-1}}\right), \qquad \text{where} \quad \begin{array}{c} S_i \text{ - asset price for } i\,th \text{ period,} \\ \text{namely currency exchange rate} \end{array}$$

and compute their standard deviations σ, using exponential weights with empirically established parameters λ and T:

$$\sigma = \sqrt{(1-\lambda) \cdot \sum_{i=2}^{T} \lambda^{i-2}(r_i - r_{avg})^2}, \quad \text{where} \qquad \begin{array}{c} \lambda - \text{decay factor, established as .89} \\ T - \text{ the number of prices used in computation,} \\ \text{established as 50} \end{array}$$

As logarithmic returns are assumed to be normally distributed, simple correction via average value and standard deviation transforms their distribution into standard normal *N(0,1)*, which in return makes it possible to calculate confidence intervals and estimate future rate changes. Further operations are as follows.

The value of the most recent return r_n is taken as a base and the next value is expected to be the same. Constructed confidence intervals then show how far the actual values may vary from the estimated with preselected probability α:

$$P\left(-t \leq \frac{r_n - r_{avg}}{\sigma \cdot \sqrt{i}} \leq t\right) = \alpha$$

As forecast is made for twenty four moments (hours) ahead, i designates the i*th* next moment and square root is included in denominator to adjust the value of σ. With that the estimated interval for r_n is

$$-t \cdot \sigma \cdot \sqrt{i} + r_{avg} \leq\ r_n \leq\ t \cdot \sigma \cdot \sqrt{i} + r_{avg}$$

Value of the unknown variable t can be estimated from the table for normal distribution probability density function. For practical reasons considered probabilities were established as 70, 80 and 95%, therefore the t values are 1.04, 1.29 and 1.96, respectively.

Using the formula for calculating logarithmic returns we can now make the same estimation for future prices:

$$-t \cdot \sigma \cdot \sqrt{i} + r_{avg} \leq\ \ln\left(\frac{S_{n+i}}{S_n}\right) \leq\ t \cdot \sigma \cdot \sqrt{i} + r_{avg}$$

$$\exp\left(-t \cdot \sigma \cdot \sqrt{i} + r_{avg}\right) \cdot S_n \leq S_{n+i} \leq \exp\left(t \cdot \sigma \cdot \sqrt{i} + r_{avg}\right) \cdot S_n$$

The result is then visualized and represented as an envelope of probable values on a graph of exchange rates.

It must be said that even 95% interval not always includes the actual value. The reason for occurring inaccuracy lies in models inability to foresee sharp dips or rallies in the future deviations. Therefore it is best to assess the situation by considering multiple characteristics and indicators apart from confidence interval.

Due to similarities in approach between the confidence interval examined herein and Bollinger Bands, it is advised to use results of computations in the same manner, namely using upper and lower limits of an interval as possible points of reversal or highs and lows for the respective periods.