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RESEARCH PRODUCTS

Economic Research

Rescaled Range Analysis

Rescaled range analysis, or RRA, is one of the many techniques used to examine and forecast the movements of economic indicators and financial assets. In contrast to various other methods, RRA can be applied to all kinds of data series regardless of their statistical properties. It can be viewed as a major advantage, as in most cases financial data does not meet the strict requirements of forecasting models.

In this research we examine the implications of RRA and the ways how to use the information it gives in practice.

In finance, *RRA* is used to study mean reversion and mean aversion in the process of price development. It is more commonly applied to stock market, but methodologically there are no restrictions against employing it for investigating exchange rates. Therefore we use *RRA* to try and classify the behaviour of two major currency pairs – EUR/USD and USD/JPY, - and two crosses – EUR/CHF and GBP/JPY. The datasets are formed from ten minute, one hour and one day candles up to August 2013.

Methodology

The main purpose of rescaled range analysis of a process $\xi(\tau)$ is to estimate its Hurst exponent – a measure of the so called long-term memory. The notion of the Hurst exponent comes from an idea about dependence between certain characteristics of the process and a time span over which they are observed.

$$\mathbb{E}[R(\tau)/S(\tau)] = (C \cdot \tau)^H, \tau \to \infty,$$

where $\mathbb{E}$ is the probabilistic average, C is a constant, and

$$R(\tau) = \max_{1 \le t \le \tau} X(t,\tau) - \min_{1 \le t \le \tau} X(t,\tau),$$

$$S(\tau) = \sqrt{\frac{1}{\tau} \sum_{t=1}^{\tau} (\xi(t) - \bar{\xi}_\tau)^2}$$

$$X(t,\tau) = \sum_{i=1}^{t} (\xi(i) - \bar{\xi}_\tau) = \sum_{i=1}^{t} \xi(i) - \frac{t}{\tau} \sum_{t=1}^{\tau} \xi(t),$$

$$\bar{\xi}_\tau = \frac{1}{\tau} \sum_{t=1}^{\tau} \xi(t)$$

Parameter $S(\tau)$ is a common standard deviation, while $X(t,\tau)$ is a deviation from the mean, accumulated over time $t \le \tau$. $X(t,\tau)$ can also be viewed as the deviation of the cumulative values of the process over t time periods from an average value for this number of periods. $R(\tau)$ is therefore described as a degree of deviation of cumulative values from their mean over time span τ. The quotient $R(\tau)/S(\tau)$ is merely a normalization of $R(\tau)$,

aimed, among other things, at producing a dimensionless quantity.

Taking the logarithm of the above formula yields the following result:

$$\log_2 \mathbb{E}[R(\tau)/S(\tau)] = H \cdot \log_2 \tau + \log_2 C, \qquad \tau \to \infty$$

As in practice we can only deal with finite time spans τ, a method has to be devised to approximate the value of H. One such method is the rescaled range analysis that is based on the idea of a simple linear regression.

A simple linear regression is a method for expressing the relationship between dependent variable y and independent variable x as a linear equation:

$$y = a \cdot x + c$$

The aim of the linear regression analysis is to estimate the unknown parameters a and c. One of the indicators of quality of the regression equation is R^2. It measures how well the equation explains the behaviour of the dependent variable. It ranges from 0 to 1.
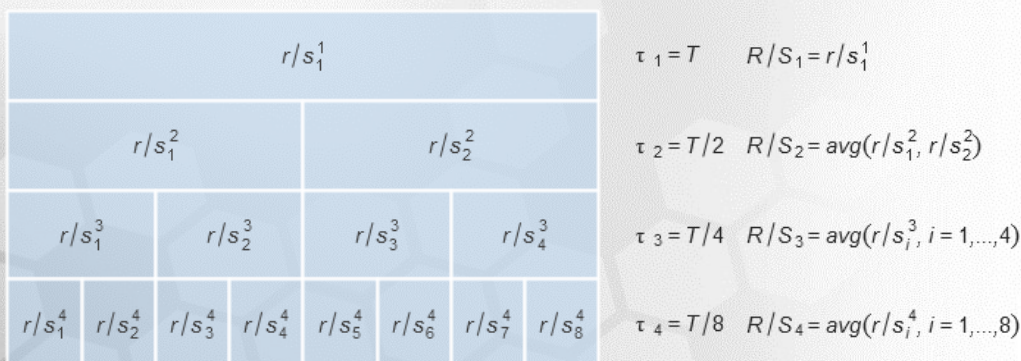

$$\xi_t = \xi_{t-1} + \varepsilon, \quad H = 0.95$$

$$\xi_t = -\xi_{t-1} + \varepsilon, \quad H = 0.1$$
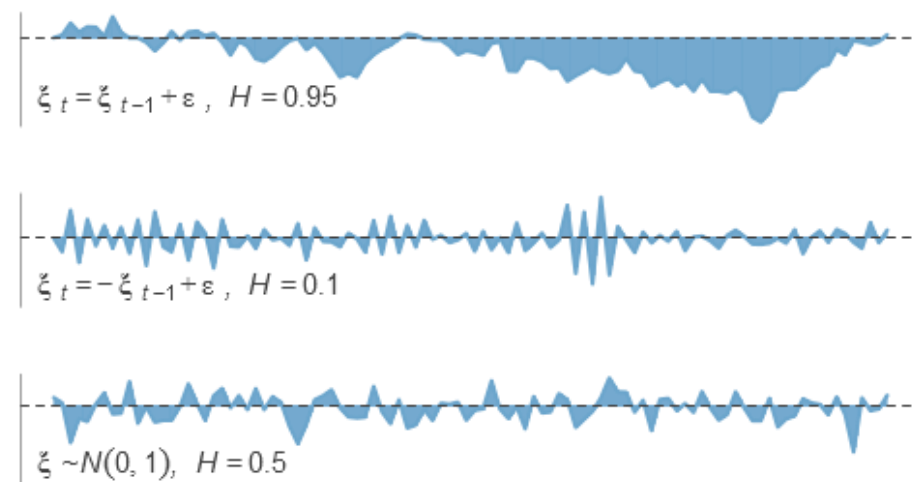
$$\xi \sim N(0, 1), \quad H = 0.5$$

Figure 1: Mean-averting, mean-reverting and random processes. Their Hurst exponents

In case of *RRA*, y and x are $\log_2(R(\tau)/S(\tau))$ and $\log_2 \tau$, but the Hurst exponent plays the role of the unknown coefficient a.

To be able to perform the linear regression analysis, we need a set of values for both $\log_2(R(\tau)/S(\tau))$ and $\log_2 \tau$. In *RRA* this is achieved by taking a set of values the process have taken over some period of time and consecutively dividing it into subsets as shown in Figure 2. The regression is then built using the values $(R/S)_i$, t_i, $i = 1,...,n$.

r/s_1^1		$\tau_1 = T \quad R/S_1 = r/s_1^1$
r/s_1^2	r/s_2^2	$\tau_2 = T/2 \quad R/S_2 = avg(r/s_1^2, r/s_2^2)$
$r/s_1^3 \quad r/s_2^3 \quad r/s_3^3 \quad r/s_4^3$		$\tau_3 = T/4 \quad R/S_3 = avg(r/s_i^3, i = 1,...,4)$
$r/s_1^4 \, r/s_2^4 \, r/s_3^4 \, r/s_4^4 \, r/s_5^4 \, r/s_6^4 \, r/s_7^4 \, r/s_8^4$		$\tau_4 = T/8 \quad R/S_4 = avg(r/s_i^4, i = 1,...,8)$

Figure 2: Dividing data into subsets for RRA

The last step of the rescaled range analysis is categorizing the process by the obtained value of H. As suggested by Figure 1, processes with H greater than 0.5 tend to follow long-term trends. That is, if the price was increasing during the analysed period, it is likely to increase during the future period of the same length. In contrast, processes with the Hurst exponent less than 0.5 are considered to be anti-persistent. A price on such an instrument tends to move back to some long-term mean, and the period of its growth is likely to be followed by the period of decline. The Hurst exponent of 0.5 is believed to indicate a random process that has no distinct behavioural patterns. The observations in such a process are independent.

Despite the fact that RRA is most commonly used in its original form, some studies show that it tends to overestimate the value of the Hurst exponent and classify random processes as persistent ones. To see if such inaccuracy really takes place, we employ a modification to RRA that is said to fix the issue. The modification consist of defining a new variable, $\log_2 H_t$, and using it instead of $\log_2 (R/S)(t)$, in regression analysis.

$$\log_2 H_t = \log_2(R/S)(t) - \log_2 F(t) + \log_2 t / 2$$, where

$$F(t) = \begin{cases} \dfrac{t-0.5}{t} \cdot \dfrac{\Gamma\left(\frac{t-1}{2}\right)}{\sqrt{\pi}\,\Gamma\left(\frac{t}{2}\right)} \displaystyle\sum_{i=1}^{t-1}\sqrt{\dfrac{t-i}{i}}, & t \le 340 \\[3ex] \dfrac{t-0.5}{t} \cdot \dfrac{1}{\sqrt{t\frac{\pi}{2}}} \displaystyle\sum_{i=1}^{t-1}\sqrt{\dfrac{t-i}{i}}, & t > 340 \end{cases} \quad , \quad \Gamma(t) = (t-1)!$$

To see if the theoretical classification corresponds to the actual behaviour of the exchange rates, we check what trends they followed during the examined periods. We determine the direction of trends by constructing the least-squares trend lines, and the strength of trends by calculating the values of ADX indicator.

Results

1. Modified *RRA* gives more accurate results for random processes.

The modification of *RRA* was originally developed to improve the estimation of the Hurst exponent for random processes. Therefore we test the methods on randomly generated normal distributions of different lengths.

Figure 3 shows that, while classical *RRA* on average overestimates the Hurst exponent, its modified version produces a slight underestimation. Such tendencies can also bias the results if the long-term memory of the process is not manifested strongly enough to overpower them. Therefore it might be useful to carry out a more complex analysis if the estimated Hurst exponent is in the range of 0.4-0.6.

Figure 3: The relationship between the estimated values of the Hurst exponent of a random process and the number or data points

On average, however, the modified *RRA* proves to be more precise as it gives a smaller root-mean-square error than its classical counterpart – *0.107* against *0.132*.

It seems that the analysis of larger sets provides a more accurate estimate of the Hurst exponent. However, processing large amounts of data is time consuming. Moreover, it requires a long exchange rate history, which might not always be available. Therefore, as the results do not seem to differ dramatically for the sets with more than a thousand elements, for future analysis we set the maximal length of datasets to be 1024.

To have a benchmark for future results we calculate the Hurst exponents for simulated data with known structure. The results are shown in Table 1. Here ε denotes a random component.
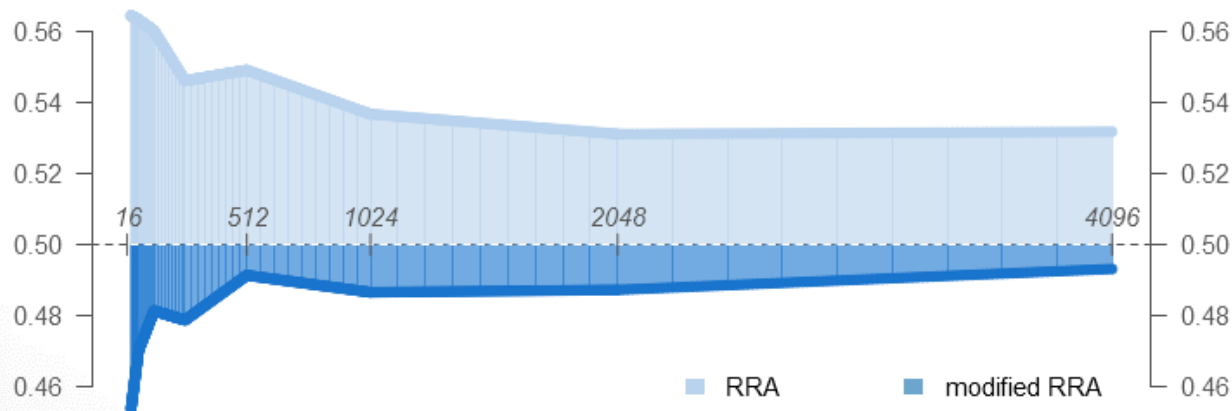
$\xi_t =$	$\xi_{t-1} + \varepsilon$	$-\xi_{t-1} + \varepsilon$	$0.6\,\xi_{t-1} + \varepsilon$	$-0.6\,\xi_{t-1} + \varepsilon$	$0.1\,\xi_{t-1} + \varepsilon$	$-0.1\,\xi_{t-1} + \varepsilon$	$\xi_{t-2} + \varepsilon$
RRA	1.02	0.06	0.63	0.45	0.49	0.57	0.85
m. RRA	0.96	0.01	0.57	0.39	0.44	0.52	0.79

Table 1: . Autoregressive processes and their estimated Hurst exponents

2. RRA classifies all examined exchange rates as strongly trending.

We have performed both classical and modified *RRA* on 500 sets consisting of 1024 close prices. Table 2 summarises the results by providing some basic descriptive statistics.

All the estimated values of the Hurst exponent are greater than 0.9, which indicates that the examined currency pairs have a tendency to follow long-term trends. It is also important that the regressions used in estimation have exceptionally high R-squared values. It suggests that the prices tend to evolve with an even speed and in general have relatively minor fluctuations along the trend line.

According to the benchmark values discussed before, such high results reflect a strong direct relationship between neighboring prices. That, of course, is not a new result. It is a common practice to model the prices of financial instruments as so-called autoregressive processes, in which every new value linearly depends on the previous one.

However, crosses are sometimes referred to as range-bound currency pairs, so it is somewhat surprising that *RRA* does not classify them as mean-reverting. We have additionally examined these pairs with weekly prices, thus covering a broader timeframe, but received the same results. That is why it seems necessary to study the real behaviour of the exchange rates within the chosen periods and see if it fits the *RRA* classification.

	10-minute					1-hour					1-day				
	CI_-	avg	CI_+	st.dev.	R^2	CI_-	avg	CI_+	st.dev.	R^2	CI_-	avg	CI_+	st.dev.	R^2
EUR/USD	1.03	1.03	1.04	0.01	0.999	1.01	1.01	1.01	0.01	0.999	0.93	0.94	0.94	0.03	0.99
	0.98	0.98	0.98	0.01	0.999	0.96	0.96	0.96	0.01	0.998	0.88	0.88	0.89	0.03	0.991
USD/JPY	1.03	1.03	1.03	0.01	0.999	1	1	1	0.01	0.999	1.02	1.02	1.02	0.01	0.998
	0.98	0.98	0.98	0.01	0.997	0.95	0.95	0.95	0.01	0.998	0.97	0.97	0.97	0.01	0.997
GBP/JPY	1.01	1.01	1.01	0.01	0.998	0.96	0.97	0.97	0.03	0.995	0.99	0.99	0.99	0.02	0.999
	0.96	0.96	0.96	0.01	0.996	0.91	0.92	0.92	0.03	0.996	0.94	0.94	0.94	0.02	0.998
EUR/CHF	1.01	1.01	1.01	0.02	0.999	0.98	0.98	0.98	0.01	0.999	1.02	1.02	1.02	0.01	0.998
	0.96	0.96	0.96	0.02	0.998	0.93	0.93	0.93	0.01	0.999	0.97	0.97	0.97	0.01	0.997

Table 2: Statistical parameters for the results of RRA. "avg" denotes the average value of the Hurst exponent, "st.dev." – the standard deviation of the value, but "CI-" and "CI+" are the lower and the upper bounds of the 95% confidence intervals

To do that we look at 25 sets of 1024 close prices and calculate an average ADX(14,9) for each set. We then check if the high values of the Hurst exponent correspond to the high values of ADX. We also use only the modified version of *RRA* from this moment henceforth, as it proved to be more accurate for high values of the Hurst exponent.

All the observed average values of ADX range from 30 to 35, indicating moderately strong trends. This matches the values of the Hurst exponent that remain above 0.9 throughout the whole examined period. In addition, the individual values of ADX rise above 25 in over 60% cases, which also supports the implications of *RRA* results.

	10-minute			1-hour			1-day		
	ADX	corr	%	ADX	corr	%	ADX	corr	%
EUR/USD	31.44	0.19	66.59	34.45	0.25	72.15	34.83	0.85	72.37
USD/JPY	32.41	-0.11	67.54	33.46	-0.55	66.68	35.63	0.21	71.91
GBP/JPY	33.3	-0.01	66.85	31.59	-0.44	58.81	33.9	0.14	72.83
EUR/CHF	31.95	0.46	69.13	35.49	0.61	74.72	35.02	0.91	76.9

Table 3: Comparison of the results of RRA and values of ADX. "corr" denotes correlation between Hurst exponent and ADX, "%" – percentage of cases, in which the values of ADX were above 25

On the downside, there seems to be no clear picture of correlation between the values of ADX and the Hurst exponent. The coefficients vary in both significance and signs. One possible reason for such ambiguity is that the values of the Hurst exponent change within a range that is too narrow to allow for clear linear dependence. It can also mean that ADX defines the strength of a trend in terms that are not considered by *RRA*.

We also check if the high value of the Hurst exponent guaranties that the direction and the strength of an existing trend will last during a period in future, as the theory suggests.

Fairly good and frequency-dependent results are observed here. Ten-minute exchange rates with high values of the Hurst exponent appear to be unreliable in pursuing a clear linear trend,

	10-minute			1-hour			1-day		
	corr	% direction	% ADX	corr	% direction	% ADX	corr	% direction	% ADX
EUR/USD	0.72	40	100	-0.2	52	48	-0.34	84	100
USD/JPY	-0.18	36	88	-0.59	68	64	0.51	72	100
GBP/JPY	0.84	36	84	0.53	72	16	0.61	40	88
EUR/CHF	-0.55	4	60	0.84	16	32	0.38	76	60

Table 4: The relationship between current value of the Hurst exponent and future behaviour of the exchange rates. "corr" denotes correlation between Hurst exponent and future ADX, "% direction" is percentage of cases, in which the direction of the current trend coincides with the direction of the future movement, but "% ADX" – in which both current and future ADX are above 25.

but they seem to preserve general strength of directed movement in over 60% of cases. Daily candles, on the other hand, keep both linearity and strength most of the time. There is, however, only one pair – GBP/JPY, - that offers consistent results regarding correlation between current value of the Hurst exponent and the future level of ADX.

Even though rescaled range analysis of exchange rates does provide some insight into their behaviour, it does not offer much new information and in general goes in line with well-known financial assumptions. Therefore it seems more interesting to use *RRA* on currency pair returns that are often believed to be purely random.

3. RRA shows a direct relationship between time-separated returns.

To examine logarithmic returns, we used the same type of analysis we carried out for close prices. Namely, we averaged the resultant values of the Hurst exponent after applying the modified *RRA* to 500 sets of 1024 returns. The results varied from 0.44 to 0.52 for all currency pairs and all frequencies, indicating the randomness of the process. However, the results for logarithmic returns, calculated over multiple exchange rate values, showed a completely different picture.

We define an *n*-period logarithmic return as $\ln \dfrac{rate(t+n)}{rate(n)}$ and use the same method for calculating average values of the Hurst exponent.

It appeared that the values of the Hurst exponent for *n*-period returns increase with an increasing *n*. That is, the multiperiod returns are not independent and tend to follow moderately strong trends.
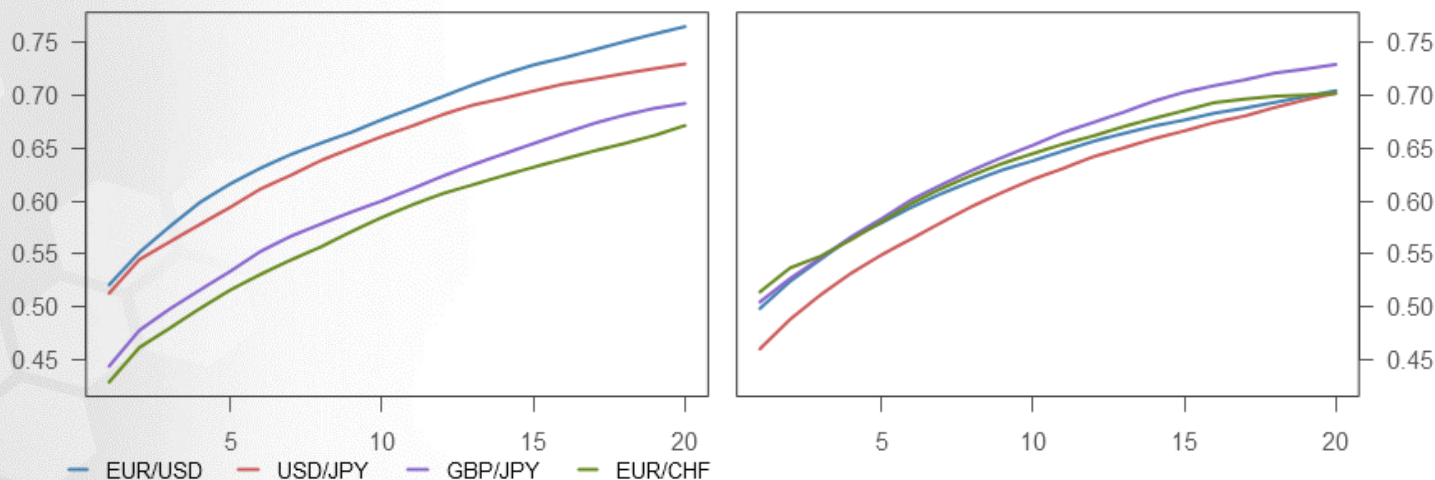

Figure 4: The relationship between the number of periods n and the Hurst exponent of the multiperiod returns by currency pair: a) one period equals one hour; b) one period equals one day

It also appears that the Hurst exponent in this case is frequency-dependent. For EUR/USD and USD/JPY lower frequencies seem to bring the values down, while for GBP/JPY the situation is the opposite. In addition, Figure 4 shows that on higher frequencies GBP/JPY and EUR/CHF have lower values of the Hurst exponent than EUR/USD and USD/JPY. It might indicate that there is, after all, a difference in the behaviour of major currency pairs and crosses.
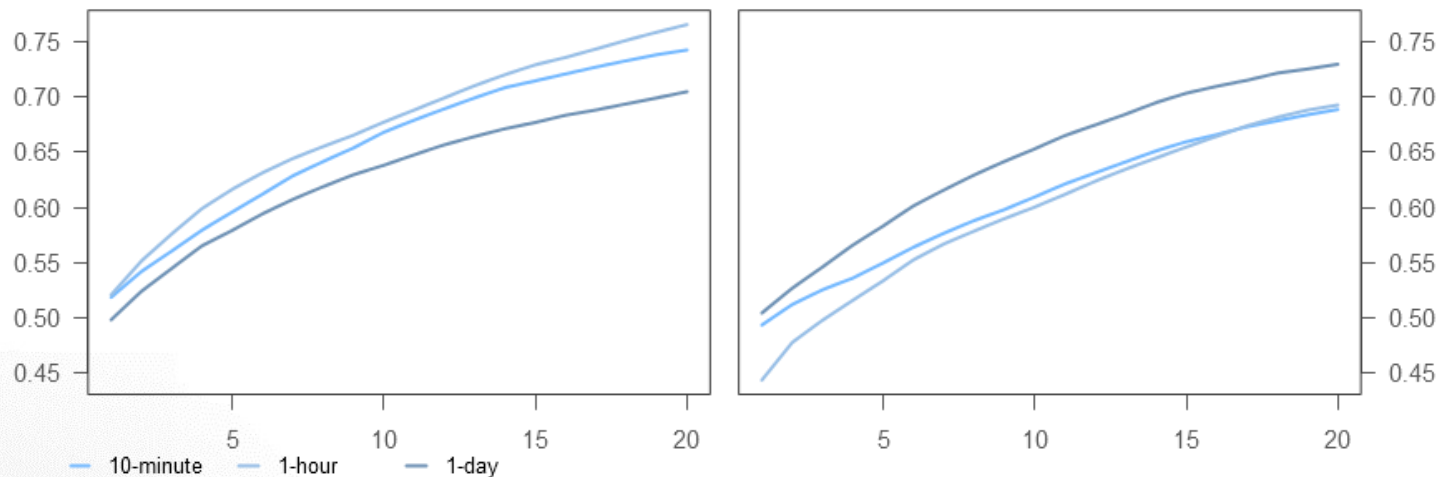

Figure 5: The relationship between the number of periods n and the Hurst exponent of the multiperiod returns by frequency: a) EUR/USD; b) GBP/JPY

Studies show, however, that the Hurst exponent can differ from 0.5 due to some exterior factors not connected to the long-memory effect. The method to check for such a possibility is to repeat *RRA* for scrambled data – a set in which the observations change places and are no longer consequent. If the resultant values of the Hurst exponent are close to the ones obtained from the original sets, the process is random. But if the new values range around 0.5, the initial assumptions about the long-memory in the data are true.

We have carried out this additional test for the multiperiod logarithmic returns. All the obtained values varied from 0.4 to 0.57, and there seemed to be no relationship between the Hurst exponent and frequency or the number of periods. This proves that the high values of the Hurst exponent for the multiperiod returns are not coincident and reflect the true nature of the process.

Conclusions

We studied two methods of estimating the Hurst exponent – the measure of long-term memory in a process. We saw that classical rescaled range analysis, which is one of the most commonly used means for estimating the parameter, and its modified version both have their own flaws. **Classical *RRA* proved to overestimate the Hurst exponent, while the modified method, on the contrary, produced underestimated values. However, modified *RRA* gave a lesser error of estimation, and thus seemed to be more accurate.**

All examined exchange rate had high values of the Hurst exponent, indicating a tendency to follow persistent trends. In most cases these high values were supported by relatively high ADX. We also saw that against the background of the high values of the Hurst exponent the existing trends most often preserve their direction and strength for some period in future. **However, the analysis of the exchange rates did not give any values of the Hurst exponent equal to or lesser than 0.5, while there were plenty of cases of the small values of ADX. Therefore it is hard to make assumptions about a relationship between the results of *RRA* and more popular measures of describing trends.**

RRA provided some new insight into the behaviour of currency pair returns. **It appeared that there is a direct relationship between multiperiod logarithmic returns, and that it strengthens with increasing periods.** Results show that the sets of ten- and more period returns have a Hurst exponent high enough to classify them as persistent. This denies the pure randomness of the development of currency pair returns and suggests that there might be fairly efficient methods for modelling the returns and thus forecasting future moves.

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