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Economic Research

Optimal Level of Leverage

In Forex, leverage is a loan that allows an investor to trade a significantly larger amount of currency than they deposit. The use of leverage is appealing, because it can help traders to increase their capital faster than while trading without it. The downside, however, is that possible losses grow together with possible gains. Therefore it is both interesting and useful to find an optimal level of leverage that would provide a compromise between increasing gains and minimising losses.

In this research we use maximisation of the end of period wealth (or, equivalently, capital rate of growth) as a criterion for optimising the level of leverage. We employ the method for the four most traded currency pairs: EUR/USD, GBP/USD, AUD/USD, and NZD/USD, and cover various time intervals for the first quarter of 2013. The data consist of one hour, ten minute, and five minute close prices.

We also test the obtained values by simulating the trading process and try to find the best way to use optimal leverage in practice.

Methodology [1]

To maximise the value of wealth at the end of a trading period one can try to maximise the “speed” of trading, or the amount gained per unit of time. One of the possible estimations of such speed is the leveraged geometric mean of returns.

$$GM(l) = \prod_{i=1}^n (1 + lr_t)^{\frac{1}{n}} - 1 \quad (1)$$

Here r_t is a logarithmic, or continuously compounded, historical return at the time moment t , n is the number of observed moments, and l is the level of leverage.

Under the assumption of normality of currency pair returns the geometric mean can be expressed in terms of simple arithmetic average of returns, μ , and the estimation of sample variance, σ^2 .

$$GM(l) = \left(l\mu - l^2 \frac{\sigma^2}{2} \right) \quad (2)$$

As can be seen from Figure 1, the last formula defines a parabolic function that has only one maximum. It yields a simple analytical expression describing the relationship between the optimal level of leverage and the parameters of the series of historical returns.

$$l_{optimal} = \frac{\mu}{\sigma^2}$$

However, in practice the assumption of normality does not always hold, mainly due to so called fat tails – extreme values of returns that appear more frequently than in a normal distribution. Therefore an analytically derived optimal level of leverage can be biased.

In order to avoid the possible bias one can use numerical methods for maximising the geometric mean given directly by equation (1). The only disadvantages of such an approach are relative computational complexity and rounding errors that are inevitable when working with computer programs. In this research we employ both analytical and numerical methods to see whether the difference in the results is great enough to choose one method over the other.

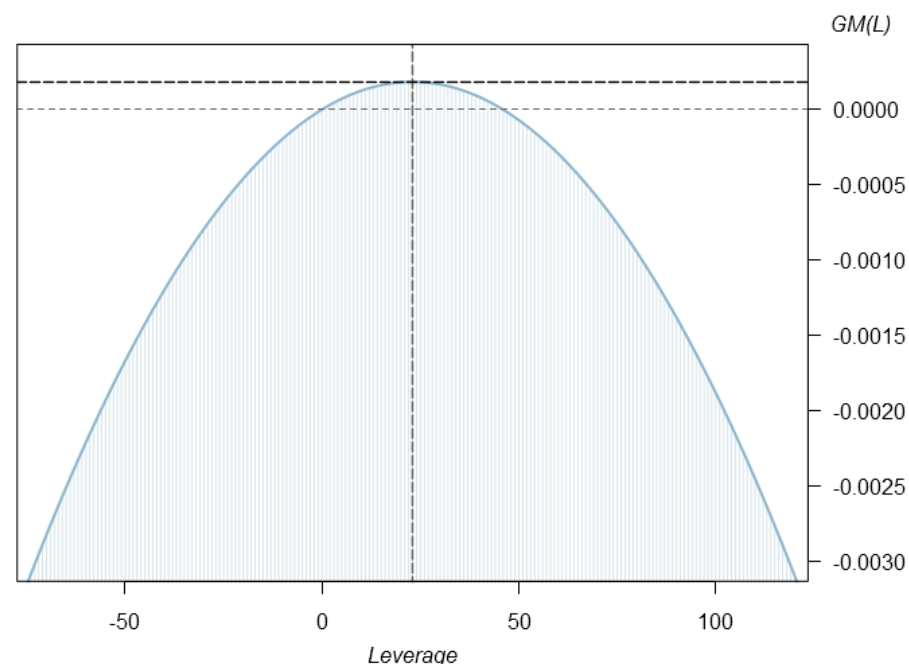


Figure 1: The relationship between the leveraged geometric mean of returns and the level of leverage for GBP/USD hourly close prices.

The proposed methods for calculating the optimal level of leverage are based on historical returns, and therefore help to establish the required level for the past trades. It is, however, interesting to see if the received values can be used to optimise future trading. To find the answer to this question we simulate trading activity with different applications of the past optimal levels of leverage.

In the first approach, we calculate analytical and numerical optimal levels of leverage over a period of 50 time moments. The moments are associated with the chosen data frequencies, and can therefore be one hour, ten minutes or five minutes long. We then use the resultant levels of leverage to trade during the next 100 moments. We denote these methods by *GBM* and *GNM*, respectively.

The second approach explores the idea of recalculating the optimal level of leverage for each new trade by shifting the window and using the latest time moments. These methods are denoted by *GBM in sequence* and *GNM in sequence*.

As both analytical and numerical optimal levels of leverage are established from a purely mathematical point of view, it is often advised to use not whole, but half the values in practice. This is said to provide more realistic levels of leverage. We investigate this theory for both constant analytical and numerical values, and values calculated in sequence.

Finally, we simulate trading with a constant level of leverage of 1:10 to see if the proposed methods can outperform trading with an arbitrary chosen level of leverage.

In the simulation itself we assume a starting balance of 100,000 USD and randomly choose the moments of opening positions. A new position gets opened in the next time moment after closing of the old position.

In trading simulation we always use the absolute value of the received level of leverage. Theoretically, the sign of the derived leverage can indicate whether a long or a short position should be opened. However, as the calculating process is based on the past returns, this information might be outdated and imprecise for future trades. Therefore we use an exponential moving average as the basic indicator for the trading strategy. Namely, we open a short position if the difference between two consequent values of the moving average is negative, and a long position otherwise.

Results

1. For all observed datasets, returns are not normally distributed.

As normality of returns is vital for the analytical method for calculating the optimal level of leverage, we started our study by performing a statistical test to see if the hypothesis holds.

A statistical test is a procedure for making decisions about a population based on a sample. The statement under study (e.g. *returns are normally distributed*) is called a null hypothesis. A significance level is a probability of making a mistake and rejecting the null hypothesis when it is true. A p-value is a numerical product received after performing the test. If the p-value is greater than the significance level, there are no grounds for rejecting the null hypothesis. Otherwise, the null hypothesis must be rejected.

	1 hour				10 min				5 min			
	EUR/USD	GBP/USD	AUD/USD	NZD/USD	EUR/USD	GBP/USD	AUD/USD	NZD/USD	EUR/USD	GBP/USD	AUD/USD	NZD/USD
p - value	6.24e-12	2.24e-11	0.00162	1.7e-06	0.00107	1.84e-13	7.55e-07	0.0019	6.6e-07	1.12e-17	4.04e-13	0.00347

Table 1: P-values of normality tests for currency pair returns.

Table 1 shows the resultant p-values of the Shapiro-Wilk test for normality. As can be seen, all the values are so small that the hypothesis of normality can be rejected on any reasonable significance level. Thus for all observed currency pairs and frequencies, returns are not normally distributed.

This means that, strictly speaking, the analytical method cannot be used for the chosen datasets. It is, however, interesting to see what kind of error the false assumption brings into the calculation. Therefore we still calculate the analytical optimal levels of leverage and compare them with numerical results.

2. Analytical and numerical methods produce very similar optimal levels of leverage.

Table 2 shows the optimal levels of leverage calculated over one window of 50 return values. The results produced by analytical and numerical methods differ in no more than thousandths, which means that for practical use they can be considered the same. Moreover, using the numerical method several times and taking the average value yields exactly the same result as the analytical approach. Therefore the violation of the analytical methods condition does not affect the result dramatically, and the method is still usable.

	1 hour				10 min				5 min			
	EUR/USD	GBP/USD	AUD/USD	NZD/USD	EUR/USD	GBP/USD	AUD/USD	NZD/USD	EUR/USD	GBP/USD	AUD/USD	NZD/USD
GBM	120.5746	222.6493	145.3674	185.1932	232.3142	8.1373	600.6957	426.5396	9.6451	120.8610	1503.1592	580.2783
GNM	120.5749	222.6516	145.3705	185.1925	232.3142	8.1373	600.6926	426.5389	9.6451	120.8610	1503.1574	580.2780

Table 2: P-values of normality tests for currency pair returns.

Another thing that can be seen from the table is that the optimal levels of leverage tend to differ not only between currency pairs, but also depending on the used frequency. There also seems to be no pattern in these differences, which suggests that the optimal levels of leverage can change rather chaotically, and might need to be monitored and recalculated.

Moreover, most values exceed 100, which is often offered as a maximal level of leverage. This points out the reason for using the half of the received values. Figure 2 shows that such approach also gives a positive value of the geometric mean, and therefore is still effective.

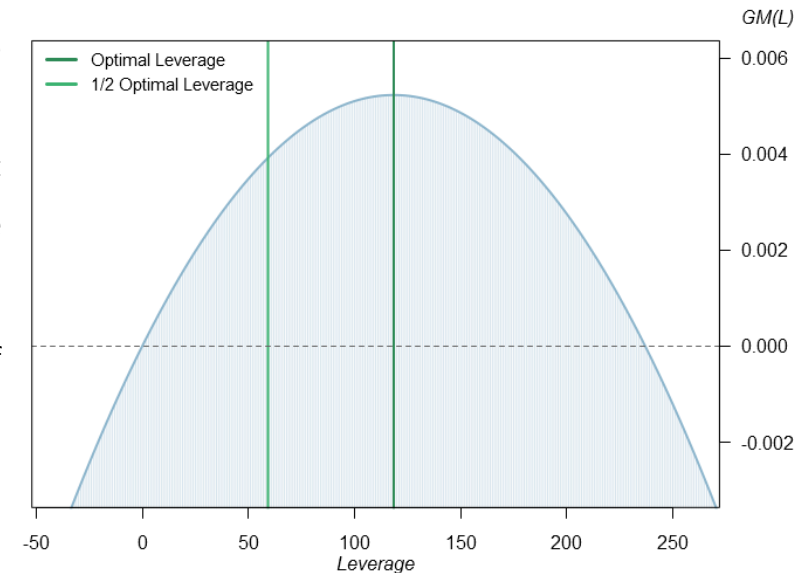


Figure 3: Optimal level and half the optimal level of leverage for GBP/USD hourly close prices.

3. None of the analysed methods is steadily better than others.

Table 3 displays average gains (or losses) of 1000 trading simulations that assumed 100,000 USD as a starting capital and used the whole optimal level of leverage. None of the methods gives stably positive results nor guaranties both maximal gains and minimal losses. Using constant GBM or GNM, for example, yields the highest gains in the case of a positive outcome, but also the biggest losses otherwise.

Table 4 gives the same information about using only half the advised levels of leverage. The results show both cases in which reducing the level of leverage leads to improving the outcome, and cases in which it has an opposite effect. Interestingly, this approach often results in increasing the losses, even though it is based on using a smaller level of leverage.

	1 hour				10 min				5 min			
	EUR/USD	GBP/USD	AUD/USD	NZD/USD	EUR/USD	GBP/USD	AUD/USD	NZD/USD	EUR/USD	GBP/USD	AUD/USD	NZD/USD
GBM	148605.90	-108490.09	-8068.82	-48691.05	-57040.08	-26453.85	26052.75	25580.76	-5750.03	3034.15	-256760.85	-71833.34
GNM	148605.82	-108490.08	-8068.83	-48691.05	-57040.08	-26453.85	26052.74	25580.76	-5750.03	3034.14	-256760.85	-71833.34
Fixed leverage 1:10	12844.12	728.21	-224.00	-3790.96	-3244.98	-1308.02	446.10	744.78	-473.93	606.41	-1776.17	-1397.51
GBM in sequence	63476.90	-14095.94	-2679.86	-28440.48	-32799.37	-13962.90	12843.09	13524.09	-2645.57	1544.10	-128521.51	-39182.83
GNM in sequence	63476.88	-14095.94	-2679.86	-28440.48	-32799.37	-13962.89	12843.09	13524.09	-2645.57	1544.10	-128521.51	-39182.83

Table 3: Average gains and losses for trading simulations with the whole optimal level of leverage.

	1 hour				10 min				5 min			
	EUR/USD	GBP/USD	AUD/USD	NZD/USD	EUR/USD	GBP/USD	AUD/USD	NZD/USD	EUR/USD	GBP/USD	AUD/USD	NZD/USD
GBM/2	107073.59	-147859.23	-65698.45	-40413.26	-45814.11	-55549.58	13572.37	-1728.90	-44341.88	-14646.95	-222729.31	-66062.13
GNM/2	107073.61	-147859.25	-65698.45	-40413.26	-45814.11	-55549.58	13572.37	-1728.90	-44341.88	-14646.94	-222729.31	-66062.13
Fixed leverage 1:10	12844.12	728.21	-224.00	-3790.96	-3244.98	-1308.02	446.10	744.78	-473.93	606.41	-1776.17	-1397.51
GBM/2 in sequence	48514.73	-53329.55	-37580.88	-24034.89	-25186.48	-26111.66	6418.57	-910.38	-21122.43	-5943.74	-102602.04	-35114.51
GNM/2 in sequence	48514.75	-53329.56	-37580.87	-24034.90	-25186.48	-26111.66	6418.57	-910.38	-21122.43	-5943.74	-102602.04	-35114.51

Table 4: Average gains and losses for trading simulations with half the optimal level of leverage.

Unfortunately, most trading simulations resulted in losses rather than gains, which means that using the past optimal level of leverage gives little to no advantage in future trading.

Conclusion

In this research we have investigated two methods – analytical and numerical, - for calculating the optimal level of leverage by maximising the leveraged geometric mean of currency pair returns. It appeared that, even though theoretically the analytical method was not appropriate for our dataset, both methods gave practically equal results. This can be viewed as a positive outcome, because the analytical method is much simpler and easier to employ.

Unfortunately, none of the methods proved to give superior results in trading simulation. It appears that the optimal level of leverage can change so significantly in time that using a past value does not guarantee any profit in present or future. It might, however, be also related to the strategy used in the simulations. It is possible that the use of a more sophisticated decision making algorithm could result in better outcomes and give more insight on the issue of the optimal levels of leverage.

Globally, it appeared that the proposed methods for calculating the optimal level of leverage can indeed shift the geometric mean of historical leveraged returns above zero. However, the application of this historical optimal level to real-time trading remains unclear.

Appendix

[1] The approach used in this research was proposed by Elton Sbruzzi and Steve Phelps in their 2012 paper “Optimal Level of Leverage using Numerical Methods”.

Shapiro-Wilk test - a statistical test with the null hypothesis of a population being normally distributed. It is often considered one of the most powerful normality tests, particularly for small samples.



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